Readers' Forum

Comment on "Computation of Second-Order Accurate Unsteady Aerodynamic Generalized Forces"

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IN a recent paper, van Niekerk[1] has presented an interesting and useful account of the application of the weighted residual method, together with the concept of adjoint systems to the calculation of generalized forces on wings in unsteady flow. He referred to a paper by the writer[2] on reverse flow and variational theorems in wing theory for a proof that the reverse flow satisfies equations adjoint to the direct forward flow equations and also states, correctly, that in that paper the reasons for the imposition of the Kutte condition on the adjoint pressure at subsonic trailing edges in reverse flow, aside from the need to make the solutions to the adjoint problem unique, were not given.

However, for the purpose of establishing the reverse flow and variational theorems of Ref. 2, it was only required to show that the imposition of the Kutte conditions to subsonic trailing edges in reverse flow was sufficient, not that it was necessary. Various mathematical methods and physical arguments have been employed[3-5] to devise reverse flow theorems in both steady and unsteady flow for the direct problems of wing theory—i.e., those in which the downwash on the wing surface is given and the velocity or pressure is to be found. All these approaches led to the imposition of a Kutte condition at subsonic trailing edges in reverse flow in order to arrive at compact, unambiguous, and meaningful results.

In an earlier paper on reverse flow and variational theorems for steady flows,[6] which also dealt with the application of the variational theorems to approximate methods in subsonic lifting surface theory analogous to those discussed by van Niekerk, the writer did address in detail the issue of necessary and sufficient conditions for defining the adjoint problem. In the latter paper, integral equation formulations of wing theory, such as those in Ref. 1, were used. It was shown that, for the integral over the wing area of pressure times downwash velocity, with the downwash velocity expressed as a singular integral of the Cauchy principal value type, the order of integration could not be reversed to lead to an adjoint equation in the manner conventionally employed in the theory of integral equations unless the Kutte condition was imposed on the adjoint (reverse flow) pressure at subsonic trailing edges. Coincidentally, the adjoint problem could then be interpreted as corresponding to the actual wing in an actual reverse flow.

Muskhelishvili[7] has given a formal mathematical discussion of the conditions to be met by direct and adjoint functions subject to integral operators of the Cauchy principal value type in order that relations equivalent to the reverse flow theorems and van Niekerk's Eq. (26) may hold. For the function sets appropriate to wing theory, application of Muskhelishvili's criteria also leads to what amounts to the Kutte condition.

Received April 21, 1986.
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References

[1] van Niekerk, B., "Computation of Second-Order Accurate Unsteady Aerodynamic Generalized Forces," *AIAA Journal*, Vol. 24, March 1986, pp. 492-498.

[2] Flax, A.H., "Reverse Flow and Variational Theorems for Lifting Surfaces in Nonstationary Compressible Flow," *Journal of the Aeronautical Sciences*, Vol. 20, Feb. 1953, pp. 120-126.

[3] Brown, C.E., "The Reversibility Theorem for Thin Airfoils in Subsonic and Supersonic Flow," NACA Rept. 986, 1950.

[4] Ursell, F. and Ward, G.N., "On Some General Theorems in the Generalized Theory of Compressible Flow," *Quarterly Journal of Mechanics and Applied Math*, Vol. III, Pt. 3, Sept. 1950, pp. 326-348.

[5] Heaslet, M.A. and Spreiter, J.R., "Reciprocity Relations in Aerodynamics," NACA Rept. 1119, 1953.

[6] Flax, A.H., "General Reverse Flow and Variational Theorems in Lifting Surface Theory," *Journal of the Aeronautical Sciences*, Vol. 19, June 1952, pp. 361-374.

[7] Muskhelishvili, N.I., *Singular Integral Equations*, Noordhoff NV, Groningen, The Netherlands, 1953, pp. 122-23, 227-29, 326.

Comment on "The Flowfield in a Suddenly Enlarged Combustion Chamber"

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IN an interesting paper,[1] Yang and Yu report a complete set of laser Doppler velocimeter (LDV) measurements on the mean velocities and Reynolds stresses for an isothermal airflow in a dump-type combustor geometry. A maximum error of 6.5% in the data, in terms of the flow rate at each measuring station, has been claimed in the paper. The detailed measurements and their projected accuracy motivated us to use them to validate our two-dimensional elliptic code STEPUP[2] for two turbulence models, the Boussinesq

Received March 10, 1986.
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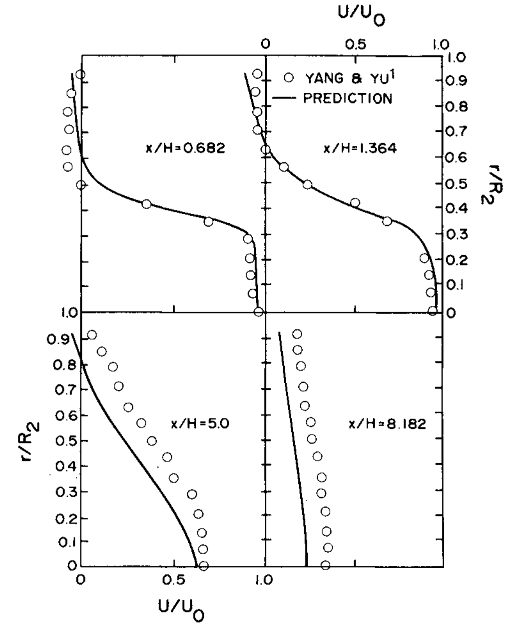

Fig. 1 Comparison of calculated mean axial velocity profiles with measurements.

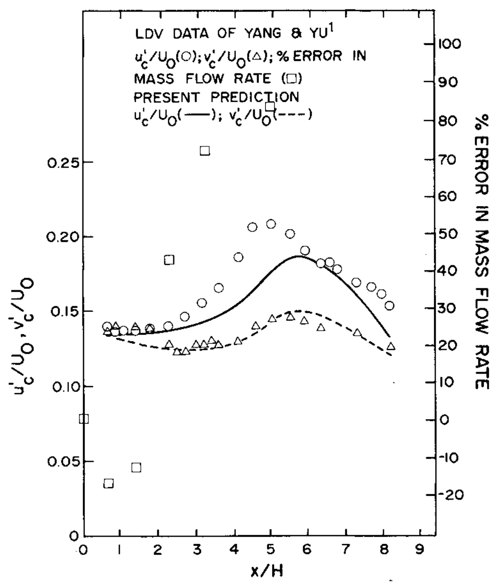

Fig. 2 Comparison of calculated axial and radial turbulence intensities with measurements at the pipe centerline, and percentage error in mass flow rate computed from measured mean axial velocity distribution.

viscosity model (BVM), or the k-ϵ model, and the algebraic stress transport model (ASM). The purpose of this comment is to communicate our experience on the standard BVM prediction of these measurements and to point out some of the serious discrepancies in the data that make them unfit for any model validation and improvement.

The flow geometry considered by Yang and Yu[1] corresponds to a dump-type combustor. In view of a short chamber length and a contraction at the outlet, the outflow does not become fully developed. For an elliptic flow computation, therefore, it becomes necessary to specify the measured profiles at the outflow boundary. Our ultimate plan for the data was to validate an algebraic stress transport model (ASM). Before attempting this model validation, the BVM simulation with the standard model constants and wall-function boundary conditions[3] was carried out.

Two main difficulties are encountered in calculating the present flow. First, the measured mean axial velocity profile at the outflow boundary ($x = 40$ cm) yields a mass flow rate that is about 65% higher than that at the inlet. Second, the measured turbulence intensity near the inlet is unusually high. In view of this, one cannot assume the commonly used inlet profiles[4] for the turbulent kinetic energy k and its dissipation rate ϵ. It has been our experience that the calculations become quite sensitive to the assumed inlet profiles of k and ϵ in the presence of high freestream turbulence. The aforementioned simulation difficulties were resolved as follows. The mean axial velocity at the outlet was reduced uniformly to satisfy continuity. At the inlet, however, along with a nearly uniform mean axial velocity, the following profile for k (using the centerline measurements of turbulence intensities reported near the expansion) was used:

$$k_{\text{in}} = 0.03 U_{\text{in}}^2 \tag{1}$$

except at the near-wall (inlet pipe) grid node, where

$$(k_{\text{in}})_p = 0.15 U_{\text{in}}^2 \tag{2}$$

was specified. The form in Eq. (2) is based on the measurements of Holladay[5] in developing pipe flow that show that the maximum value of k near the wall is about five times its value in the freestream (core flow). The inlet condition for ϵ, specified as,

$$(\epsilon_{\text{in}})_p = C_\mu^{3/4} (k_{\text{in}})_p^{3/2} / (0.41 y_p) \tag{3}$$

is based on the existence of a thin, equilibrium wall boundary layer. In Eq. (3), C_μ is the model constant equal to 0.09 and y_p is the distance of the near-wall grid node from the wall.

Using the numerical procedure presented elsewhere,[2] a reattachment length of six step heights (as opposed to the experimentally observed value of 4.5) results from the present simulation with the standard BVM. Figure 1 shows that the computed mean axial velocity profiles are in good agreement with the data near the expansion, at $x/H = 0.682$ and 1.346. Further downstream at $x/H = 5.0$ and 8.182, although the calculations agree qualitatively with the measurements, quantitatively there appears to be a nearly constant shift between the two across the chamber radius. Results for the axial and radial turbulence intensities along the chamber axis are also in satisfactory agreement with the measurements shown in Fig. 2. Since the computed mean axial velocity profiles are mass conserving, the disagreement with the data downstream of the expansion indicates that the corresponding experimental profiles (including the one at the outflow boundary) are inconsistent from a continuity standpoint. The continuity error in these experimental velocity profiles from direct numerical integration at several of the measurement stations is also shown in Fig. 2, as a percentage deviation from the inlet flow rate. The figure shows a value as high as 80% for this error, which does not agree with the reported value of only 6.5%.

Yang[6] suspects that the error might have been due to a shift in the blower output. There could be other possible sources of error in the present data set. A part of the error is suspected to be due to a velocity bias that is known to worsen in regions of high-turbulence intensities. This is, at least, qualitatively evident from the observed trends in the axial variation of flow rate error and the turbulence intensities illustrated in Fig. 2. The importance of velocity bias error has been demonstrated in a recent investigation of Stevenson et al.[7] For a sudden expansion pipe flow geometry, Stevenson et al.[8] have reported both the biased and unbiased mean axial velocity measurements using LDV. Their biased velocity data exhibit a maximum flow rate error of about 24%, whereas the maximum turbulence intensity is reported to be 20%. In the case of the present data set, however, the discrepancies found are serious enough to discourage its use for any meaningful validation and development of an advanced turbulence model.

Acknowledgment

This investigation was conducted at Arizona State University.

References

[1]Yang, B. T. and Yu, M. H., "The Flowfield in a Suddenly Enlarged Combustion Chamber," *AIAA Journal*, Vol. 21, Jan. 1983, pp. 92-97.

[2]Sultanian, B. K., "Numerical Modeling of Turbulent Swirling Flow Downstream of an Abrupt Pipe Expansion," Ph.D. Thesis, Arizona State University, Tempe, 1984.

[3]Launder, B. E. and Spalding, D. B., "The Numerical Calculation of Turbulent Flow," *Computer Methods in Applied Mechanics and Engineering*, Vol. 3, 1974, pp. 263-289.

[4]Gosman, A. D., Khalil, E. E., and Whitelaw, J. H., "The Calculation of Two-Dimensional Turbulent Recirculating Flows," *Proceedings Symposium on Turbulent Shear Flows*, edited by F. Durst et al., Springer-Verlag, New York, April 1977, pp. 237-255.

[5]Holladay, D. W., "An Experimental Investigation of the Turbulent Flow in the Entrance Region of a Smooth Round Pipe," Ph.D. Thesis, The University of Tennessee, 1968.

[6]Yang, B. T., private communications, 1983.

[7]Stevenson, W. H., Thompson, H. D., and Roesler, T. C., "Direct Measurement of Laser Velocimeter Bias Errors in a Turbulent Flow," *AIAA Journal*, Vol. 20, 1982, pp. 1720-1723.

[8]Stevenson, W. H., Thompson, H. D., and Gould, R. D., "Laser Velocimeter Measurements and Analysis in Turbulent Flows with Combustion Part II," AFWAL-TR-82-2076, Pt. II, 1982.